Volume and Surface Area Scaling

Grade Range: Middle School

Lesson Time: 60 minutes

Key Terms

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Materials and Resources

- Paper to show work

Activity Overview

Think about your childhood toys: Were any of them scale (or not-to-scale) models of vehicles, animals, or even prism-shaped structures such as buildings? The volumes and surface areas of realistic models will be proportionate to those of the full-size objects. Students will solve for the volume and surface area of rectangular and triangular prisms. They will also build prisms using a given scale factor, and determine how the scale factor affected the volume and surface area. Students will apply the Pythagorean Theorem to determine the surface area of the triangular prism models.

Essential Questions

1. How do volume and surface area scale in relation to the scale factor?
2. Why do volume and surface area scale differently?
3. Why does volume scale faster than surface area?

Objectives

- Create models of prisms with a given scale factor
- Determine the volume for prisms with a scale factor
- Explore the relationship between the original prism and the prism with a scale factor
- Recognize that the volume of a prism with a scale factor is affected by a power of three
- Determine the surface area of prisms using the Pythagorean Theorem to calculate the hypotenuse of triangles on the faces of triangular prisms
- Recognize that the surface area of a prism with a scale factor is affected by a power of two

Introduction

Prior to beginning this activity, students must know what a scale factor is, how to apply scale factors to 3D solids, and how to solve for the volume and surface area of rectangular and triangular prisms. Optionally, have students complete Volume and Surface Area of Prisms (Fractions and Decimals) in Newton’s Park to review volume and surface area of rectangular and triangular prisms in preparation for this activity.
To begin the activity, ask students to raise their hands if they like to build or collect scale models. Allow them to share briefly about the types of models they enjoy. Describe to students how architects must be able to make scale models of their designs to show at presentations. The volume and surface area of their designs must stay proportional to those of their actual design. Then, review how to solve for volume and surface area. Also, review that applying a scale factor means changing each dimension of a solid. Explain that students will apply scale factors to various solids and observe how this affects the solids’ volumes and surface areas.

**zSpace Activity**

**Activity Questions Provided in Newton’s Park**

*Answers may vary. Sample answers are provided below.*

1. What are the volume and surface area of solid A?
   
   \[ V = 13.125 \text{ or } 13 \frac{1}{8} \text{ cubic meters; } SA = 35.5 \text{ or } 35 \frac{1}{2} \text{ square meters} \]

2. Change solid A by a scale factor of 2. Take a photo. Hint: Place your scaled model on the middle of the grid board when you take your photo. Hint: Make sure you can see the dimensions, including the height, when you take your photo.
   
   Photo: \( L = 5 \text{ meters, } W = 3 \text{ meters, and } H = 7 \text{ meters.} \)

3. What are the new volume and surface area of solid A after applying the scale factor of 2?
   
   \[ V = 105 \text{ cubic meters; } SA = 142 \text{ square meters} \]

4. Place your model in the trash.

5. What are the volume and surface area of solid B?
   
   \[ V = 48 \text{ cubic meters; } SA = 88 \text{ square meters} \]

6. Change solid B by a scale factor of \( \frac{3}{4} \). Take a photo.
   
   Photo: \( L = 4.5 \text{ meters, } W = 3 \text{ meters, and } H = 1.5 \text{ meters.} \)

7. What are the new volume and surface area of solid B after applying the scale factor of \( \frac{3}{4} \)?
   
   \[ V = 20.25 \text{ or } 20 \frac{1}{4} \text{ cubic meters; } SA = 49.5 \text{ or } 49 \frac{1}{2} \text{ square meters} \]

8. Place your model in the trash.

9. What are the volume and surface area of solid C?
   
   \[ V = 18 \text{ cubic meters; } SA = 48 \text{ square meters} \]

10. Change solid C by a scale factor of \( \frac{1}{2} \). Take a photo.
    
    Photo: \( L = 2 \text{ meters, } W = 1.5 \text{ meters, and } H = 1.5 \text{ meters.} \)

11. What are the new volume and surface area of solid C after applying the scale factor of \( \frac{1}{2} \)?
    
    \[ V = 2.25 \text{ or } 2 \frac{1}{4} \text{ cubic meters; } SA = 12 \text{ square meters} \]

12. Place your model in the trash.

13. What are the volume and surface area of solid D?
    
    \[ V = 40.5 \text{ or } 40 \frac{1}{2} \text{ cubic meters; } SA = 54 \text{ square meters} \]

14. Change solid D by a scale factor of 2. Take a photo.
    
    Photo: \( L = 6 \text{ meters, } W = 1.5 \text{ meters, and } H = 4.5 \text{ meters.} \)

15. What are the new volume and surface area of solid D after applying the scale factor of 2?
    
    \[ V = 324 \text{ cubic meters; } SA = 216 \text{ square meters} \]

16. Place your model in the trash.
Closing

Questions for Discussion

1. Why, when we used a scale factor of 2, as with Solid A, was the volume increased by 8 times and the surface area by only 4 times?
   In order to solve for the volume, you have to multiply by three dimensions (L × W × H). Therefore, since each dimension is doubled the volume will change by 2 × 2 × 2 = 8 times greater. The surface area is only two dimensions (L × W). Therefore, the surface area will only change by 4 times.

2. Why did we need to use the Pythagorean Theorem to solve the surface area of the triangular prisms?
   The triangular prisms had triangles, so we needed to find the hypotenuse of the triangle in order to find the length of slanted side of the face where the ramp is.

3. Why would an architect need to use scale models?
   If he or she was making a presentation, showing the design at actual size would be impossible. The scale model would need to be proportionally accurate, though, so people could see what it would look like if it was full scale.

Extension Activity: Students could build their own prisms and then determine a scale factor that will work; they could then solve for the volume and surface area of both prisms (original and scaled)

Differentiation

- Group students heterogeneously to allow students with a strong command of the English language to assist in reading or interpreting questions
- Provide paper copies of diagrams for students to use as a reference
- Provide a handout with a list of vocabulary terms and definitions that will appear in the activity
- Allow students to provide answers that are handwritten, typed, or verbal
- Have students work as partners or in small groups
- Use text-to-speech if needed
- Enrichment: Students could find real-world problems involving the concept and design solutions to those problems
- Enrichment: Students could work on the discussion questions and lead the class discussion
- Enrichment: Students could build a model of a key concept